Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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5	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM04

Unit Mechanics 4

Monday 25 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 A light rod has its ends at the points $P(-2, -1, 4)$ and $Q(4, 1, 6)$. A force $\mathbf{F}$ acts at the point M, the mid-point of PQ, where $\mathbf{F} = a\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(a) Show that $\overrightarrow{PM} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. *(2 marks)*

(b) Find the moment of $\mathbf{F}$ about the point P, giving your answer in terms of a. *(3 marks)*

(c) Given that the magnitude of the moment is $5\sqrt{2}$, find the two possible values of a. *(4 marks)*

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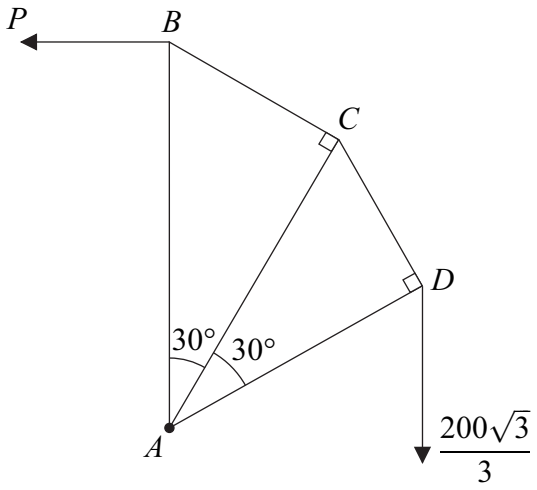
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2 A framework is composed of five light smoothly-jointed rods, AB, AC, AD, BC and CD. The framework contains two right-angled triangles, ABC and ACD. The angle $BAC =$ angle $CAD = 30°$. The lengths of AB, AC and AD are $2l$, $\sqrt{3}l$ and $\frac{3l}{2}$ respectively.

The framework is in equilibrium in a vertical plane and is freely hinged at A to a fixed support. A vertical force of $\frac{200\sqrt{3}}{3}$N acts at D. The rod AB is kept vertical by a horizontal force of magnitude P newtons, acting in the same plane as the framework at B, as shown in the diagram.



(a) Show that $P = 75$. *(3 marks)*

(b) Find the magnitudes of the forces in the rods BC and AB, stating whether each rod is in tension or compression. *(6 marks)*

(c) By considering the forces perpendicular to AC at the point C, find the magnitude of the force in the rod CD. *(2 marks)*

(d) For each of the rods CD, AC and AD, state whether the rod is in tension or compression. *(2 marks)*

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3 Four forces, acting in the x–y plane, act at the points with coordinates as listed below, where p and q are constants and **i** and **j** are parallel to the x-axis and y-axis respectively.

Force		**Coordinates**
$-2\mathbf{i}$	acting at	$(3, 3)$
$-5\mathbf{j}$	acting at	$(4, 0)$
$p\mathbf{i} + \mathbf{j}$	acting at	$(-8, -2)$
$-3\mathbf{i} - 4\mathbf{j}$	acting at	$(0, q)$

(a) Write down the resultant of these forces in terms of p. *(1 mark)*

(b) (i) Given that the system of forces is equivalent to a single force, **F**, acting parallel to the y-axis, find the value of p. *(2 marks)*

(ii) Given further that the equation of the line of action of **F** is $x = 3$, find the value of q. *(6 marks)*

(c) The system of forces can be replaced by a single force acting at the origin together with a couple C.

Write down the magnitude of C and indicate its sense on a diagram. *(2 marks)*

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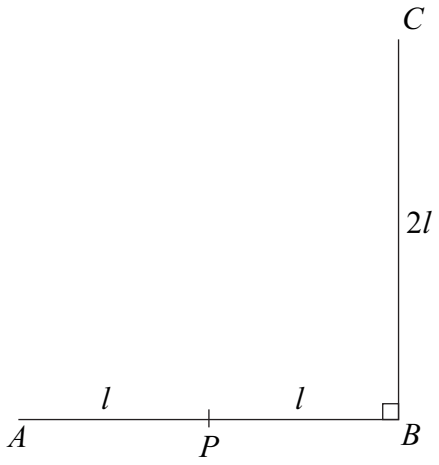
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4 Two identical uniform rods, AB and BC, are rigidly joined together to form a right angle at B. The mid-point of the rod AB is P. Each rod has mass m and length $2l$. The rods can rotate freely in a vertical plane about a horizontal axis through P, perpendicular to the plane containing ABC.



(a) Show that the moment of inertia of the rod BC about the axis through P is $\frac{7ml^2}{3}$. *(4 marks)*

(b) Particles of masses $3m$, $3m$ and $4m$ are fixed at A, B and C respectively.

Find the moment of inertia of the whole system about the axis through P. *(5 marks)*

(c) The system is released from rest with AB horizontal and C vertically above B.

Find, in terms of g and l, the angular velocity of the system when BC is horizontal and A is vertically above B. *(7 marks)*

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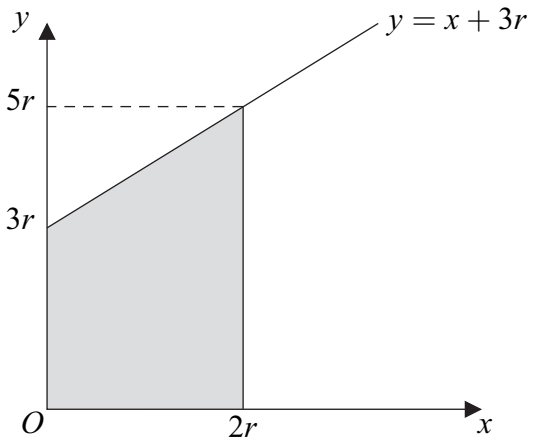
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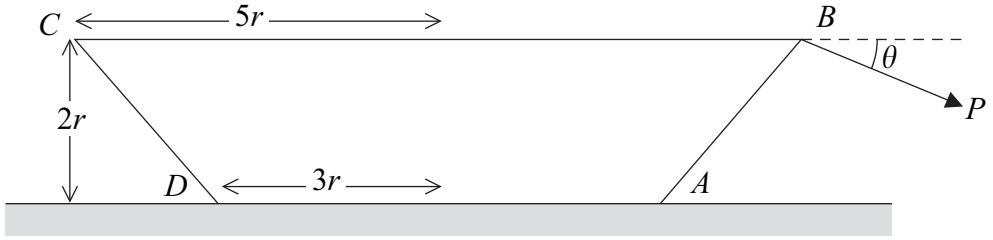
5 The region bounded by the line $y = x + 3r$, the y-axis, the x-axis and the line $x = 2r$ is shown in the diagram.



This region is rotated about the x-axis to form a frustum, of volume $\frac{98\pi r^3}{3}$, of a uniform solid cone.

(a) Using integration, find the distance of the centre of mass of the frustum of the cone from O. *(6 marks)*

(b) The frustum of a uniform solid cone with radii $3r$ and $5r$ and height $2r$ has weight W. The frustum stands on a horizontal surface. The diagram shows a cross-section, $ABCD$, which includes the axis of symmetry, of this frustum. A force P is applied to the point B and acts in the same plane as this cross-section at an angle of θ below the horizontal.



The frustum does not slide and is about to topple about A.

(i) Show that

$$P = \frac{3W}{2(\cos\theta + \sin\theta)}$$ *(4 marks)*

(ii) Find, in terms of W, the minimum possible value of P. *(3 marks)*

(iii) State the value of θ for which P is a minimum. *(1 mark)*

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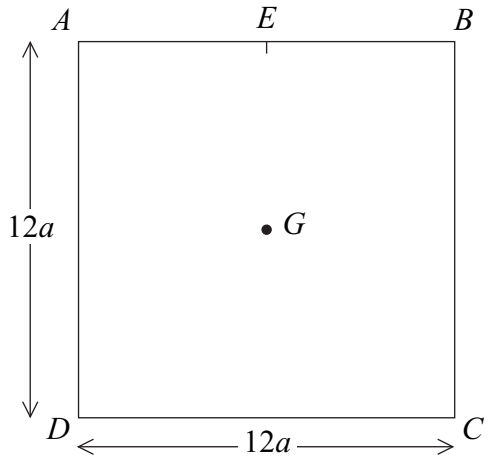
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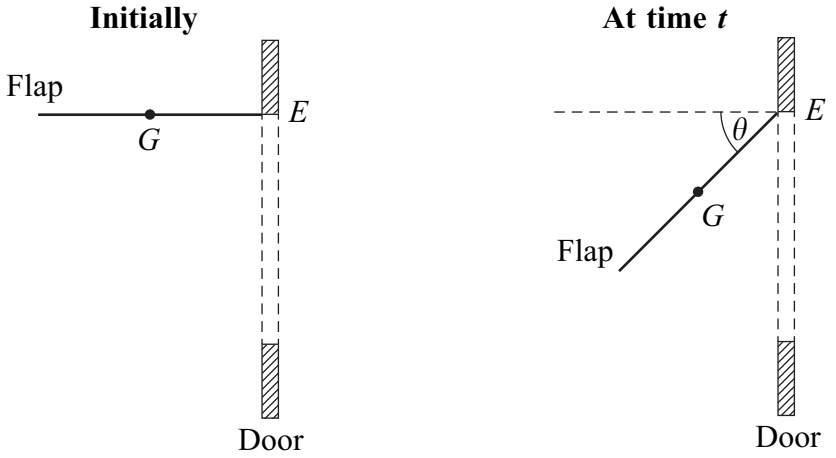
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6 A uniform lamina, of mass m, is in the shape of a square, $ABCD$, with each side of length $12a$. The centre of mass of the lamina is G, and the mid-point of AB is E.



(a) Show, by using integration, that the moment of inertia of the square lamina about a fixed horizontal axis along the side AB is $48ma^2$. *(5 marks)*

(b) A cat flap in a door can be modelled by the lamina described above. The flap is free to rotate about a fixed horizontal axis along AB. Initially, the flap is held at rest so that EG is horizontal, and it is then released. At time t after release, EG makes an angle θ with the horizontal.



(i) Show that $\ddot{\theta} = \dfrac{g\cos\theta}{8a}$. *(3 marks)*

(ii) Find, in terms of m, g and θ, the magnitude of the reaction that the flap exerts on the axis in the direction perpendicular to EG. *(3 marks)*

(iii) State one improvement that could be made to the model. *(1 mark)*

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